a) $\dfrac{(x^2-9)}{x^2-9}\dfrac{dy}{dx} + \dfrac{xy}{x^2-9} = \dfrac{0}{x^2-9}$

$$\frac{dy}{dx} + \frac{x}{x^2-9}\,y = 0$$

$$y' + \frac{x}{x^2-9}\cdot y = 0$$

$$P(x) = \frac{x}{x^2-9} \quad e \quad Q(x) = 0$$

$$I(x) = e^{\int P(x)\,dx}$$

$$I(x) = e^{\underbrace{\int \frac{x}{x^2-9}\,dx}_{\text{substituição}}}$$

$u = x^2 - 9$

$du = 2x\,dx$

$\Downarrow$

$$I(x) = e^{\int \frac{\cancel{x}}{u}\frac{du}{2\cancel{x}}}$$

$$I(x) = e^{\int \frac{1}{2}\frac{du}{u}}$$

$$I(x) = e^{\frac{1}{2}\ln u}$$

$$I(x) = e^{\frac{1}{2}\ln(x^2-9)}$$

$$I(x) = e^{\log_e (x^2-9)^{1/2}}$$

$$I(x) = (x^2-9)^{1/2}$$

$$I(x) = \sqrt{x^2-9}$$

$$I \cdot y = \int I \cdot Q(x)\,dx$$

$$\sqrt{x^2-9}\cdot y = \int \sqrt{x^2-9}\cdot 0\,dx$$

$$\sqrt{x^2-9}\cdot y = \int 0\,dx$$

$$\sqrt{x^2-9}\cdot y = c$$

$$y = \frac{c}{\sqrt{x^2-9}}$$

b) $y' + 3x^2 y = x^2$

$P(x) = 3x^2$ e $Q(x) = x^2$

$I(x) = e^{\int P(x)\,dx}$

$I(x) = e^{\int 3x^2\,dx}$

$I(x) = e^{3\int x^2\,dx}$

$I(x) = e^{\frac{\cancel{3}x^3}{\cancel{3}}}$

$I(x) = e^{x^3}$

$I.\,y = \int I.\,Q(x)\,dx$

$I.\,y = \underbrace{\int e^{x^3} x^2\,dx}_{\text{substituição}}$

$u = x^3$

$du = 3x^2\,dx$

$x^2\,dx = \frac{du}{3}$

$\int e^u \frac{du}{3}$

$\frac{1}{3}\int e^u\,du$

$\frac{1}{3}e^u + C$

$\frac{1}{3}e^{x^3} + C$

$e^{x^3}.\,y = \int e^{x^3} x^2\,dx$

$e^{x^3}.\,y = \frac{1}{3}.\,e^{x^3} + C$

$y = \frac{1}{3}.\,\frac{\cancel{e^{x^3}}}{\cancel{e^{x^3}}} + \frac{C}{e^{x^3}}$

$y = \frac{1}{3} + C.\,e^{-x^3}$

c) $\dfrac{dy}{dx} = 5y$

$y' = 5y$

$y' - 5y = 0$

$P(x) = -5$ e $Q(x) = 0$

i) $I(x) = e^{\int P(x)\,dx}$

$I(x) = e^{\int -5\,dx}$

$I(x) = e^{-5\int 1\,dx}$

$I(x) = e^{-5x}$

ii) $I.y = \int I.\,Q(x)\,dx$

$I.y = \int e^{-5x}.0$

$e^{-5x}.y = \int 0\,dx$

$e^{-5x}.y = C$

$y = \dfrac{C}{e^{-5x}}$

$y = C.e^{5x}$

---

d) $\dfrac{dy}{dx} + 2y = 0$

$y' + 2y = 0$

$P(x) = 2$ e $Q(x) = 0$

i) $I(x) = e^{\int P(x)\,dx}$

$I(x) = e^{\int 2\,dx}$

$I(x) = e^{2\int 1\,dx}$

$I(x) = e^{2x}$

ii) $I\,y = \int I\,Q(x)\,dx$

$e^{2x}.y = \int e^{2x}.0\,dx$

$y.e^{2x} = \int 0\,dx$

$y.e^{2x} = C$

$y = \dfrac{C}{e^{2x}}$

$y = C.e^{-2x}$

e) $\frac{dy}{dx} + y = e^{3x}$

$y' + y = e^{3x}$

$P(x) = 1$ e $Q(x) = e^{3x}$

$I(x) = e^{\int P(x)\,dx}$

$I(x) = e^{\int 1\,dx}$

$I(x) = e^{x}$

$I \cdot y = \int I \cdot Q(x)\,dx$

$e^{x} \cdot y = \int e^{x} \cdot e^{3x}\,dx$

$e^{x} \cdot y = \underbrace{\int e^{4x}\,dx}_{\text{substituição}}$

$u = 4x$

$du = 4\,dx$

$\int e^{4x}\,dx = \int e^{u} \cdot \frac{1}{4}\,du$

$\int e^{u} \frac{du}{4} = \frac{1}{4} e^{4x} + C$

$e^{x} y = \frac{1}{4} \cdot e^{4x} + C$

$y = \frac{1}{4} \cdot \frac{e^{4x}}{e^{x}} + \frac{C}{e^{x}}$

$y = \frac{e^{3x}}{4} + C \cdot e^{-x}$

---

f) $\frac{xy'}{x} + \frac{2y}{x} = \frac{3}{x}$

$y' + \frac{2}{x} y = \frac{3}{x}$

$P(x) = \frac{2}{x}$ e $Q(x) = \frac{3}{x}$

$I(x) = e^{\int P(x)\,dx}$

$I(x) = e^{\int \frac{2}{x}\,dx}$

$I(x) = e^{2 \int \frac{1}{x}\,dx}$

$I(x) = e^{2 \ln x}$

$I(x) = e^{\log_e x^2} = x^2$

$I \cdot y = \int I \cdot Q(x)\,dx$

$x^2 y = \int x^2 \cdot \frac{3}{x}\,dx$

$x^2 y = \int 3x\,dx$

$x^2 y = 3 \int x\,dx$

$x^2 y = 3 \frac{x^2}{2} + C$

$y = \frac{3}{2} \frac{x^2}{x^2} + \frac{C}{x^2}$

$y = \frac{3}{2} + \frac{C}{x^2}$

h) $\dfrac{x^2y'}{x^2} + \dfrac{xy}{x^2} = \dfrac{1}{x^2}$

$$y' + \frac{1}{x}y = \frac{1}{x^2}$$

$$P(x) = \frac{1}{x} \quad e \quad Q(x) = \frac{1}{x^2}$$

$$I(x) = e^{\int P(x)\,dx}$$

$$I(x) = e^{\int \frac{1}{x}dx}$$

$$I(x) = e^{\ln x}$$

$$I(x) = e^{\log_e x}$$

$$I(x) = x$$

$$I.y = \int I.Q(x)\,dx$$

$$x.y = \int x.\frac{1}{x^2}\,dx$$

$$xy = \int \frac{1}{x}\,dx$$

$$xy = \ln x + C$$

$$y = \frac{\ln x}{x} + \frac{C}{x}$$

---

g) $\dfrac{x\,dy}{x\,dx} - \dfrac{y}{x} = \dfrac{x^2 \operatorname{sen} x}{x}$

$$y' - \frac{1}{x}y = x \operatorname{sen} x$$

$$P(x) = -\frac{1}{x} \quad e \quad Q(x) = x \operatorname{sen} x$$

$$I(x) = e^{\int P(x)\,dx}$$

$$I(x) = e^{\int -\frac{1}{x}dx}$$

$$I(x) = e^{-1\int \frac{1}{x}dx}$$

$$I(x) = e^{-\ln x}$$

$$I(x) = e^{\log_e x^{-1}}$$

$$I(x) = x^{-1}$$

$$I.y = \int I(x)\,Q(x)\,dx$$

$$x^{-1}.y = \int x^{-1}.x.\operatorname{sen} x\,dx$$

$$x^{-1}y = \int \operatorname{sen} x\,dx$$

$$x^{-1}.y = -\cos x + C$$

$$y = -\frac{\cos x}{x^{-1}} + \frac{C}{x^{-1}}$$

$$y = C.x - x.\cos x$$

i) $\dfrac{dy}{dx} + y = x$

$y' + y = x$

$P(x) = 1$ e $Q(x) = x$

$I(x) = e^{\int P(x)\,dx}$

$I(x) = e^{\int 1\,dx}$

$I(x) = e^{x}$

$I.y = \int I.Q(x)\,dx$

$e^{x}y = \underbrace{\int e^{x}.x\,dx}_{\text{Partes}}$

$u = x \quad \int dv = \int e^{x}dx$

$du = dx \quad v = \int e^{x}dx$

$v = e^{x}$

$e^{x}.y = uv - \int v\,du$

$e^{x}.y = x.e^{x} - \int e^{x}dx$

$e^{x}y = xe^{x} - e^{x} + c$

$y = \dfrac{xe^{x}}{e^{x}} - \dfrac{e^{x}}{e^{x}} + \dfrac{c}{e^{x}}$

$\boxed{y = c.e^{-x} + x - 1}$

---

j) $\dfrac{xy'}{x} + \dfrac{2y}{x} = \dfrac{\operatorname{sen} x}{x}$

$y' + \dfrac{2}{x}y = \dfrac{\operatorname{sen} x}{x}$

$P(x) = \dfrac{2}{x}$ e $Q(x) = \dfrac{\operatorname{sen} x}{x}$

$I(x) = e^{\int P(x)\,dx}$

$I(x) = e^{\int \frac{2}{x}dx}$

$I(x) = e^{2\int \frac{1}{x}dx}$

$I(x) = e^{2\ln x}$

$I(x) = e^{\log_e x^2}$

$I(x) = x^2$

$I.y = \int I.Q(x)\,dx$

$x^2y = \int x^2.\dfrac{\operatorname{sen} x}{x}\,dx$

$x^2y = \underbrace{\int x.\operatorname{sen} x\,dx}_{\text{Partes}}$

$x^2y = uv - \int v\,du$

$u = x \quad \int dv = \int \operatorname{sen} x\,dx$

$du = dx \quad v = -\cos x$

$x^2y = x(-\cos x) - \int -\cos x\,dx$

$x^2y = -x\cos x + \operatorname{sen} x + c$

$y = -\dfrac{x\cos x}{x^2} + \dfrac{\operatorname{sen} x}{x^2} + \dfrac{c}{x^2}$

$\boxed{y = -\dfrac{\cos x}{x} + \dfrac{\operatorname{sen} x}{x^2} + \dfrac{c}{x^2}}$

k) $y' - y = 2 \cdot e^{2x}$

$P(x) = -1$ e $Q(x) = 2 \cdot e^{2x}$

$I(x) = e^{\int P(x)\,dx}$

$I(x) = e^{\int -1\,dx}$

$I(x) = e^{-1\int 1\,dx}$

$I(x) = e^{-x}$

$I \cdot y = \int I \cdot Q(x)\,dx$

$I \cdot y = \int e^{-x} \cdot 2e^{2x}\,dx$

$e^{-x} \cdot y = \int 2 \cdot e^{x}\,dx$

$e^{-x} y = 2\int e^{x}\,dx$

$e^{-x} y = 2 \cdot e^{x} + C$

$y = \dfrac{2e^{x}}{e^{-x}} + \dfrac{C}{e^{-x}}$

$y = 2 \cdot e^{2x} + C \cdot e^{x}$

---

l) $\dfrac{x^2 y'}{x^2} + \dfrac{2xy}{x^2} = \dfrac{\cos x}{x^2}$

$y' + \dfrac{2}{x} y = \dfrac{\cos x}{x^2}$

$P(x) = \dfrac{2}{x}$ e $Q(x) = \dfrac{\cos x}{x^2}$

$I(x) = e^{\int P(x)\,dx}$

$I(x) = e^{\int \frac{2}{x}\,dx}$

$I(x) = e^{2\int \frac{1}{x}\,dx}$

$I(x) = e^{2\ln x}$

$I(x) = e^{\log_e x^2}$

$I(x) = x^2$

$I \cdot y = \int I \cdot Q(x)\,dx$

$x^2 \cdot y = \int \cancel{x^2} \cdot \dfrac{\cos x}{\cancel{x^2}}\,dx$

$x^2 \cdot y = \int \cos x\,dx$

$x^2 y = \operatorname{sen} x + C$

$y = \dfrac{\operatorname{sen} x}{x^2} + \dfrac{C}{x^2}$

m) $y' - 2xy = x$

$P(x) = -2x$ e $Q(x) = x$

$I(x) = e^{\int P(x)\,dx}$

$I(x) = e^{\int -2x\,dx}$

$I(x) = e^{-2\int x\,dx}$

$I(x) = e^{-\frac{\cancel{2}x^2}{\cancel{2}}}$

$I(x) = e^{-x^2}$

$I \cdot y = \int I \cdot Q(x)\,dx$

$e^{-x^2} \cdot y = \underbrace{\int e^{-x^2} \cdot x\,dx}_{\text{substituição}}$

$u = -x^2$

$du = -2x\,dx$

$x\,dx = \frac{du}{-2}$

$\int e^u \frac{du}{-2} = -\frac{1}{2}\int e^u du$

$= -\frac{1}{2} e^{-x^2} + C$

$e^{-x^2} \cdot y = -\frac{1}{2} e^{-x^2} + \frac{C}{}$

$y = -\frac{1}{2} \frac{\cancel{e^{-x^2}}}{\cancel{e^{-x^2}}} + \frac{C}{e^{-x^2}}$

$y = -\frac{1}{2} + C \cdot e^{x^2}$

02) a) $\begin{cases} y' - y = 2x \cdot e^{2x} \\ y(0) = 1 \end{cases}$

$P(x) = -1$ e $Q(x) = 2x \cdot e^{2x}$

$I(x) = e^{\int P(x)\,dx}$

$I(x) = e^{\int -1\,dx}$

$I(x) = e^{-1\int 1\,dx}$

$I(x) = e^{-x}$

$I \cdot y = \int I \cdot Q(x)\,dx$

$e^{-x} \cdot y = \int e^{-x} \cdot x \cdot 2 \cdot e^{2x}\,dx$

$e^{-x} y = \underbrace{\int 2x \cdot e^{x}\,dx}_{\text{Partes}}$

$u = 2x \qquad \int dv = \int e^{x}\,dx$

$du = 2\,dx \qquad v = \int e^{x}\,dx$

$v = e^{x}$

$e^{-x} \cdot y = 2x \cdot e^{x} - \int e^{x} \cdot 2\,dx$

$e^{-x} \cdot y = 2x\,e^{x} - 2e^{x} + C$

$y = \dfrac{2x \cdot e^{x}}{e^{-x}} - \dfrac{2 \cdot e^{x}}{e^{-x}} + \dfrac{C}{e^{-x}}$

$\underbrace{y = 2x \cdot e^{2x} - 2 \cdot e^{2x} + C \cdot e^{x}}_{\text{Solução Geral}}$

p/ $y(0) = 1$.

$1 = 2 \cdot 0 \cdot e^{0} - 2 \cdot e^{0} + C \cdot e^{0}$

$1 = -2 + C$

$\underline{\underline{C = 3}}$

$y = 2x \cdot e^{2x} - 2e^{2x} + 3e^{x}$

$y = e^{x}(2x e^{x} - 2 \cdot e^{x} + 3)$

$\underbrace{y = e^{x} \cdot [2e^{x}(x-1) + 3]}_{\text{Solução Particular}}$

02) b) $\begin{cases} y' + 2y = x \cdot e^{-2x} \\ y(1) = 0 \end{cases}$

$P(x) = 2$ e $Q(x) = x \cdot e^{-2x}$

$I(x) = e^{\int 2\,dx}$

$I(x) = e^{2\int 1\,dx}$

$I(x) = e^{2x}$

$I \cdot y = \int I \cdot Q(x)\,dx$

$I\,y = \int e^{2x} \cdot x \cdot e^{-2x}\,dx$

$e^{2x} \cdot y = \int x\,dx$

$e^{2x} \cdot y = \dfrac{x^2}{2} + c$

$y = \dfrac{x^2}{2 \cdot e^{2x}} + \dfrac{c}{e^{2x}}$

$y = \dfrac{x^2 \cdot e^{-2x}}{2} + c \cdot e^{-2x}$

Solução Geral

P/ $y(1) = 0$

$0 = \dfrac{e^{-2}}{2} + c \cdot e^{-2}$

$-\dfrac{\cancel{e^{-2}}}{2} = c \cdot \cancel{e^{-2}}$

$c = -\dfrac{1}{2}$

$y = \dfrac{x^2 \cdot e^{-2x}}{2} - \dfrac{1}{2} \cdot e^{-2x}$

$y = \dfrac{1}{2} \cdot e^{-2x}\left(x^2 - 1\right)$

Solução Particular

e) $\begin{cases} y' = x + y \\ y(0) = 1 \end{cases}$

$y' - y = x$

$P(x) = -1$ e $Q(x) = x$

$I(x) = e^{\int P(x)\,dx}$

$I(x) = e^{\int -1\,dx}$

$I(x) = e^{-1\int 1\,dx}$

$I(x) = e^{-x}$

$I . y = \int I . Q(x)\,dx$

$e^{-x} . y = \underbrace{\int e^{-x} x\,dx}_{\text{Partes}}$

$u = x$ e $\int dv = \int e^{-x}\,dx$

$du = dx$ $\quad v = \underbrace{\int e^{-x}\,dx}_{\text{substituição}}$

$u = -x$

$du = -dx$

$v = \int e^{u} \frac{du}{-1}$

$v = -1 \int e^{u}\,du$

$v = -e^{-x}$

$e^{-x} . y = x . (-e^{-x}) - \int -e^{-x} (+dx)$

$e^{-x} . y = -x . e^{-x} + \int e^{-x}\,dx$

$e^{-x} . y = -x . e^{-x} - e^{-x} + C$

$y = \frac{-x . e^{-x}}{e^{-x}} - \frac{e^{-x}}{e^{-x}} + \frac{C}{e^{-x}}$

$y = \underbrace{-x - 1 + C . e^{x}}_{\text{Solução Geral}}$

P/ $y(0) = 1$

$1 = 0 - 1 + C . e^{0}$

$C = 2$

$y = -x - 1 + 2e^{x}$

Solução Particular

d) $\begin{cases} x^2y' + 2xy = \cos x \\ y(\pi) = 0 \end{cases}$

$$\frac{x^2y'}{x^2} + \frac{2xy}{x^2} = \frac{\cos x}{x^2}$$

$$y' + \frac{2}{x}y = \frac{\cos x}{x^2}$$

$$P(x) = \frac{2}{x} \quad e \quad Q(x) = \frac{\cos x}{x^2}$$

$$I(x) = e^{\int P(x)\,dx}$$

$$I(x) = e^{\int \frac{2}{x}\,dx}$$

$$I(x) = e^{2\int \frac{1}{x}\,dx}$$

$$I(x) = e^{2\ln x}$$

$$I(x) = e^{\log_e x^2}$$

$$I(x) = x^2$$

$$I \cdot y = \int I \cdot Q(x)\,dx$$

$$x^2 y = \int \cancel{x^2} \cdot \frac{\cos x}{\cancel{x^2}}\,dx$$

$$x^2 y = \int \cos x\,dx$$

$$x^2 y = \operatorname{sen} x + C$$

$$\underbrace{y = \frac{\operatorname{sen} x}{x^2} + \frac{C}{x^2}}_{\text{Solução Geral}}$$

P/ $y(\pi) = 0$

$$\frac{\operatorname{sen} \pi}{\pi^2} + \frac{C}{\pi^2} = 0$$

$$\frac{C}{\pi^2} = 0$$

$$C = 0$$

$$y = \frac{\operatorname{sen} x}{x^2} + \frac{0}{x^2}$$

$$\underbrace{y = \frac{\operatorname{sen} x}{x^2}}_{\text{Solução Particular:}}$$